DICK SCHAFF SUPERBOWL XLVII

2022 Junior High School Bomb Exam – Page 1 of 5

School _____ Team _____

Directions: **(1) Label answers with appropriate units.**
(2) Do not round or approximate answers.
(3) Write fully simplified answers on the lines provided.

1. If Katia rolls two standard six-sided dice 600 times, how many times can she expect to roll a sum of 4?

Student Name: _____ Ans: _____

2. The *star* pictured below is a regular pentagram. What is the degree measure inside each point (or tip) of the *star*?

Student Name: _____ Ans: _____

3. How many 4-digit natural numbers have a ten's digit greater than the one's digit?

Student Name: _____ Ans: _____

4. The hot water tap can fill a bathtub in 4 minutes. The cold water tap can fill a bathtub in 3 minutes. The drain can empty the entire bathtub in 6 minutes. If both taps are turned on and the drain is open, how long will it take the water in the bathtub to begin to overflow?

Student Name: _____ Ans: _____

5. How many composite numbers are there from 40 to 80 inclusive?

Student Name: _____ Ans: _____

School _____ Team _____

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1. The average of six numbers is 39. Three of the numbers are 27, 38, and 31. The remaining three numbers all have the same value. Find the value.

Student Name: _____ Ans: _____

2. Evaluate $1-2+3-4+5-6+\cdots+2019-2020$

Student Name: _____ Ans: _____

3. What digit is in the one's place of 27^{2020} ?

Student Name: _____ Ans: _____

4. Find the value of n, given that $\dfrac{(n+1)!}{(n-1)!}=20$

Student Name: _____ Ans: _____

5. A sequence contains common fractions in which the denominator is increased and then the numerator is increased as shown below. What is the 5050^{th} fraction in this sequence?

$$\frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{4},\ \frac{2}{4},\ \frac{3}{4},\ \frac{1}{5},\ \frac{2}{5},\ \frac{3}{5},\ \frac{4}{5},\ \frac{1}{6},\ \frac{2}{6},\ \frac{3}{6},\ \frac{4}{6},\ \frac{5}{6},\ \frac{1}{7},\ \cdots$$

Student Name: _____ Ans: _____

School _____ Team _____

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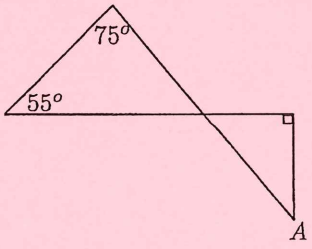
1. Sixty pet-owners were surveyed and it was found that 32 owned dogs, 24 owned cats, and 20 owned birds. Three owned all three types of pets, and 3 owned none of these types of animals. Five people owned dogs and cats. Ten people owned dogs and birds. How many people owned cats and birds?

Student Name: _____ Ans: _____

2. Two vertices of a rectangle are $(-3, -2)$ and $(1, -2)$. The line with the equation $y = -1$ is a line of symmetry for this rectangle. How many units long is the perimeter of this rectangle?

Student Name: _____ Ans: _____

3. Find the measure of the angle inside the triangle at the vertex marked A.



Student Name: _____ Ans: _____

4. Given that $2^y = 16^{x-3}$ and $3^{y+2} = 27^x$ find the value of $x + y$.

Student Name: _____ Ans: _____

5. Find the last digit of the following sum: $1! + 2! + 3! + 4! + \cdots + 2019! + 2020!$.

Student Name: _____ Ans: _____

School _____ Team _____

Directions: **(1) Label answers with appropriate units.**
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1. If a can of soda sells for \$1.50 and the can itself costs \$1.00 more than the contents, what is the value of the contents?

Student Name: _____ Ans: _____

2. A number divided by three more than itself gives a quotient of 0.4. What is the number?

Student Name: _____ Ans: _____

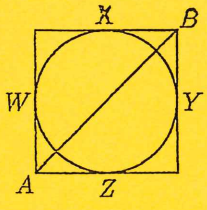
3. Find the exact value of $(123456789)(123456789) - (123456786)(123456792)$

Student Name: _____ Ans: _____

4. Line L has equation $y = -\frac{1}{2}x + 4$. Line P has the same x-intercept as L and is also perpendicular to L. As an ordered pair, what is the y-intercept of line P?

Student Name: _____ Ans: _____

5. In the diagram, the circle is inscribed in the square, meaning the circle and the square share points W, X, Y, and Z. If the square has side length 1, determine the total length of the two pieces of segments AB that are outside the circle.



Student Name: _____ Ans: _____

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Directions: **(1) Label answers with appropriate units.**
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1. What is the 1000^{th} decimal digit of 0.12345678910111213145161718192021...?

Student Name: _____ Ans: _____

2. How many squares of any size are on an 8×8 chess board?

Student Name: _____ Ans: _____

3. Let $\mathbb{S}$ be the set consisting of the number 3142 together with all other different 4-digit numbers formed by rearranging the digits of 3142. Find the sum of the elements of set $\mathbb{S}$.

Student Name: _____ Ans: _____

4. Find the value of $4^{3^{2^0}}$

Student Name: _____ Ans: _____

5. *Modular arithmetic* is also called *clock arithmetic* because it can be thought of as arithmetic using the whole numbers on the face of a clock. For example, on a standard clock the equation $11 + 2 = 1$ is true. With a standard clock we would say we were working in *mod* 12. If we are working with a clock having only 7 numbers on its face, we say we are working in *mod* 7, and we have only the numbers $\{1, 2, 3, 4, 5, 6, 7\}$ to work with. In *mod* 7, what is $1 \div 3$?

Student Name: _____ Ans: _____